$\sqrt{\ }$ Algorithm – Sheet #1

1) State the two laws of repeating decimals.

2) Convert each fraction into an exact decimal.

a) $\frac{19}{54}$

b) $\frac{19}{125}$

c) $\frac{19}{26}$

3) Calculate.

a) $\sqrt{36}$

b) $\sqrt{3600}$

c) $\sqrt{360000}$

d) $\sqrt{36000000}$

e) $\sqrt{144}$

f) $\sqrt{14400}$

g) $\sqrt{1440000}$

h) $\sqrt{144000000}$

i) $\sqrt{490000}$

j) $\sqrt{900}$

k) $\sqrt{250000}$

l) $\sqrt{40000}$

m) $\sqrt{4000000}$

4) Look at the previous problems in order to answer the following questions. (Assume that all square roots work out to whole numbers.)

a) If a number has 3 digits, then its square root will have _____ digits.

b) If a number has 4 digits, then its square root will have _____ digits.

c) If a number has 5 digits, then its square root will have _____ digits.

d) If a number has 6 digits, then its square root will have _____ digits.

e) If a number has 7 digits, then its square root will have _____ digits.

f) If a number has 8 digits, then its square root will have _____ digits.

g) If a number has 25 digits, then its square root will have _____ digits.

h) If a number has 26 digits, then its square root will have _____ digits.

5) Considering the previous answers, give a general law that states how many digits the answer for any square root problem will have.

6) Calculate.

a) 20^2

b) 90^2

c) 400^2

d) 300^2

e) 7000^2

f) 1100^2

g) 80000^2

h) 634^2

7) If a number has 3 digits, then squaring it will give a number with _____ digits.

8) Considering the above answer, give a general law that states how many digits the answer for squaring a number will have.

The Trial and Error Method.

This method for calculating square roots is quite easy to understand but not very efficient. We simply *guess* what the answer might be, and then *check* how good our guess was by squaring it and seeing if it was too big or too small.

Example: Find $\sqrt{2209}$.

Solution: We might first guess that the answer is 50, so we check that by squaring 50, which is 2500, and since 2500 is greater than 2209, we know that $\sqrt{2209}$ must be less than 50. Our next guess may be 42, so we check it by squaring 42, which is 1764, telling us $\sqrt{2209}$ must be greater than 42. Similarly, we might try 45 (which turns out to be too small) and 48 (which is too big), until we finally narrow the answer down to 47, which is exactly correct since 47^2 is equal to 2209.

Also, if we know for sure that the square root works out exactly, then the last digit inside the square root can give us a clue to the answer. For example, with $\sqrt{2209}$, since the last digit inside the square root is 9, we know that the last digit in our answer must be a 3 or a 7. This is because only 3^2 (=<u>9</u>) and 7^2 (=4<u>9</u>) end with a digit of 9. Again, this only works if we know for sure that the square root works out evenly.

9) Calculate. (The answers work out exactly.)

a) $\sqrt{1521}$

b) $\sqrt{4624}$

c) $\sqrt{74529}$

$\sqrt{\ }$ Algorithm – Sheet #2

1) Calculate.

a) $\sqrt{16000000}$

b) $\sqrt{90000}$

c) $\sqrt{1210000}$

d) 600^2

e) 15000^2

2) *For each of the below problems*, state the number of digits that the answer will have (before the decimal point), and state what the first digit will be.

Do not calculate the square root exactly.

Example: $\sqrt{467856}$

Solution: $\sqrt{467856}$ has 3 digits and the first digit is 6.

Note: It turns out that $\sqrt{467856}$ is equal to 684 (which you don't have to calculate).

a) $\sqrt{2601}$ has ___ digits; The first digit is ___.

b) $\sqrt{537289}$ has ___ digits; The first digit is ___.

c) $\sqrt{1369}$ has ___ digits; The first digit is ___.

d) $\sqrt{79524}$ has ___ digits; The first digit is ___.

e) $\sqrt{74390625}$ has ___digits; The first digit is ___.

f) $\sqrt{88209}$ has ___ digits; The first digit is ___.

3) Using formulas.

Galileo's Law of Falling Bodies.

$$D = 16 \cdot T^2$$

This formula tells us something about dropping rocks off cliffs. Specifically, it calculates how many feet the rock will fall (D) after being in the air for T seconds. (Note: it assumes zero air resistance.)

a) How far does a rock fall after being dropped from a cliff for 1 second?

b) How far does a rock fall after being dropped from a cliff for 2 seconds?

c) How far does a rock fall after being dropped from a cliff for 3 seconds?

d) How far does a rock fall after being dropped from a cliff for 5 seconds?

e) How far does a rock fall after being dropped from a cliff for 10 seconds?

f) How high would a cliff need to be in order for a rock to be able to fall for a whole minute before hitting the ground?

4) Using formulas.

Heron's Formula for the Area of a Triangle.

Area $= \sqrt{s(s-a)(s-b)(s-c)}$
Where s is the semi-perimeter (half the perimeter).

Example: Find the area of the triangle with sides of length 5, 6, and 7 inches.

Solution: We set a,b,c equal to 5,6,7. The perimeter is 18, so s is set to 9, which gives us:

$$\text{Area} = \sqrt{9(2)(3)(4)} = \sqrt{216} \approx \underline{14.7\text{in}^2}$$

a) Find the area of the triangle with sides of length 6, 8, and 10 meters.

b) Find the area of the triangle with sides of length 13, 14, and 15 feet.

c) Find the area of the triangle with sides of length 3, 5, and 6 centimeters.

5) Using formulas.

The Squaring Formula.

$$(a+b)^2 = a^2 + b(2a+b)$$

This formula gives us a very different method for calculating the square of a number. It says that if I want to calculate the square of a number (call it N), then I can first break it down into two parts, a and b, such that a + b = N, and then put these values for a and b into the formula in order to get my final answer.

Example: Calculate 17^2 using the Squaring Formula.

Solution: We can choose any two numbers that add to 17, but 10 and 7 are the easiest. Putting these numbers into the formula gives us:

$$\begin{aligned} 17^2 \rightarrow (10+7)^2 &= 10^2 + 7(2\cdot 10 + 7) \\ &= 100 + 7(27) \\ &= 100 + 189 \\ &= 289 \end{aligned}$$

(which is equal to 17^2)

Note: There are many other possible squaring formulas. This particular one does not save us time for calculating the square of a number. It is, in fact, much easier to get an answer by simply multiplying the number times itself, as we would normally do. *The reason that we are using this formula is that it will be of great use for us when we learn a method for calculating square roots – something called the "Square Root Algorithm".*

a) Calculate 26^2 by using the Squaring Formula.

b) Calculate 83^2 by using the Squaring Formula.

c) Calculate 74^2 by using the Squaring Formula.

d) Calculate 39^2 by using the Squaring Formula.

$\sqrt{}$ Algorithm – Sheet #3

1) For each of the below problems, state the number of digits that the answer will have (before the decimal point), and the first digit of the answer. *Do not calculate what the square root is equal to.* (If you need help, then look at the previous worksheet.)

a) $\sqrt{7384}$ has ___ digits;
The first digit is ___.

b) $\sqrt{67482}$ has ___ digits;
The first digit is ___.

c) $\sqrt{985035}$ has __ digits;
The first digit is ___.

d) $\sqrt{803}$ has ___ digits;
The first digit is ___.

e) $\sqrt{9670564}$ has __ digits;
The first digit is ___.

2) Calculate by using the Squaring Formula (see previous worksheet).

a) 57^2

b) 14^2

c) 95^2

The Long Algebraic Method (for two-digit answers)

Example: Calculate $\sqrt{6889}$.

Solution: Here n = 6889. We know that its square root has 2 digits, and that the first digit is 8 (because $\sqrt{68}$ is between 8 and 9). We call our first estimate of the answer a, and in this case a = 80. The second digit we call b.

Here is the procedure:

Using the square root identity

$$\mathbf{n - a^2 = b(2a + b)}$$

(which comes from $\sqrt{n} = a + b$)

We put in n = 6889 and a = 80 we get:

$$\begin{aligned} 6889 - 80^2 &= b(2 \cdot 80 + b) \\ 6889 - 6400 &= b(160 + b) \\ 489 &= b(160 + b) \end{aligned}$$

Now we determine b (the answer's second digit). We try different single digit values for b to see what works.

For example,
for b = 2, then we do $16\underline{2} \cdot \underline{2}$;
for b = 5, then we do $16\underline{5} \cdot \underline{5}$,

hoping that one of them will be equal to (or just under) 489. It turns out that b = 3 works ($16\underline{3} \cdot \underline{3} = 489$). Therefore, our answer is **<u>83</u>**. (Since $163 \cdot 3$ is exactly 489, we know that our answer is exact. We can check our answer by squaring 83 to get exactly 6889.)

3) Calculate each square root using the Long Algebraic Method. It is important that you do the problem and organize your work on a separate sheet exactly like the example just given. (All answers work out exactly.)

a) Calculate $\sqrt{3249}$

b) Calculate $\sqrt{5329}$

c) Calculate $\sqrt{784}$

d) Calculate $\sqrt{8464}$

$\sqrt{}$ Algorithm – Sheet #4

1) Calculate each square root using the *Long Algebraic Method*, using the method shown on the previous worksheet. (All answers work out exactly.)

a) $\sqrt{2025}$ b) $\sqrt{361}$ c) $\sqrt{7056}$ d) $\sqrt{4356}$

The Long Algebraic Method (for larger answers)

- For square roots that have answers with more than two digits, we need to do the same procedure as above, but repeat the process a number of times.
- Keep in mind that the a values are the digits that we are certain of at a given point, and the b values are the digits that we are trying to figure out.
- **Notation:** a_1 means the value of a (with one correct digit) for the first time through the process.
  a_2 means the value of a (with two correct digits) for the second time through the process.
  a_3 means the value of a (with three correct digit) for the third time through the process.
  The values of b are similarly given as b_1, b_2, b_3, etc.

Example: Calculate $\sqrt{7203856}$.

Step #1 We know the answer has 4 digits, and the first digit is 2 (because $\sqrt{7}$ is between 2 and 3), so $a_1 = 2000$, and we use the identity $n - a_1^2 = b_1(2a_1 + b_1)$, where $2a_1 = 4000$.

$$\begin{array}{lrl} \mathbf{n} & 7203856 & \\ \mathbf{a_1^2} & \underline{-4000000} & \text{(because } 2000^2 = 4000000) \\ \mathbf{n-a_1^2} & 3203856 & = b_1(4000 + b_1) \text{, where } b_1 \text{ is the 100's place (e.g. 300, 400, etc.)} \end{array}$$

$\underline{b_1 = 600}$ because 700 is too big, which means $b_1(2a_1 + b_1) = 2760000$

Step #2 We now know that the first two digits are 2 and 6, so $a_2 = 2600$, and we use the identity $n - a_2^2 = b_2(2a_2 + b_2)$, where $2a_2 = 5200$.

$$\begin{array}{lrl} \mathbf{n} & 7203856 & \\ \mathbf{a_2^2} & \underline{-6760000} & \text{(because } 2600^2 = 6760000) \\ \mathbf{n-a_2^2} & 443856 & = b_2(5200 + b_2) \text{, where } b_2 \text{ is the tens' place (e.g. 30, 40, etc.)} \end{array}$$

$\underline{b_2 = 80}$ because 90 is too big, which means $b_2(2a_2 + b_2) = 422400$

Step #3 We now know that the first three digits are 2, 6, and 8, so $a_3 = 2680$, and we use the identity $n - a_3^2 = b_3(2a_3 + b_3)$, where $2a_3 = 5360$.

$$\begin{array}{lrl} \mathbf{n} & 7203856 & \\ \mathbf{a_3^2} & \underline{-7182400} & \text{(because } 2680^2 = 7182400) \\ \mathbf{n-a_3^2} & 21456 & = b_3(5360 + b_3) \text{, where } b_3 \text{ is the ones' place (e.g. 3, 4, etc.)} \end{array}$$

$\underline{b_3 = 4}$, which means $b_3(2a_3 + b_3) = 21456$,

which means that our final answer is *exactly* **2684**.

2) Calculate each square root using the Long Algebraic Method. It is important that you do the problem and organize your work exactly like the example given above. (All answers work out exactly.)

a) $\sqrt{285156}$ b) $\sqrt{71289}$ c) $\sqrt{524176}$ d) $\sqrt{767376}$

$\sqrt{\ }$ Algorithm – Sheet #5

Calculate each square root using the Long Algebraic Method. It is important that you do the problem and organize your work exactly like the example given on the previous worksheet. (All answers work out exactly.)

1) $\sqrt{184041}$

2) $\sqrt{128164}$

3) $\sqrt{34596}$

4) $\sqrt{5593225}$

5) $\sqrt{72131049}$

√‾ Algorithm – Sheet #6

1) Calculate each square root using the *Long Algebraic Method*, as done on the previous worksheets. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$

The Short Algebraic Method

<u>The Basic Idea</u>

- *Reducing the amount of Calculating.* The long algebraic method, described above, requires some tedious, and unnecessary, calculations, which can be eliminated.
- Look at example for the long algebraic method shown on sheet #4. Looking at the left side of each step, we see, for step #1: $n - a_1^2$, and then for step #2: $n - a_2^2$, etc.
- Since $a_2 = a_1 + b_1$, we can use the *Squaring Formula* $(a+b)^2 = a^2 + b(2a+b)$ to get:

$$a_2^2 = (a_1 + b_1)^2 = a_1^2 + b_1(2a_1 + b_1)$$

<u>This is the key idea</u>: In place of subtracting a_2^2 from n, we can instead subtract the whole of $\{a_1^2 + b_1(2a_1 + b_1)\}$ from n since it is equal to a_2^2. This seems like more work, but it's not - it's less work.

In other words, instead of doing $n - a_2^2$, we can do $n - \{a_1^2 + b_1(2a_1 + b_1)\}$,
which is the same as $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$

In short: instead of doing $n - a_2^2$ we do $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$
Likewise, instead of doing $n - a_3^2$ we do $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$
Likewise, instead of doing $n - a_4^2$ we do $(n - a_3^2) - \{b_3(2a_3 + b_3)\}$, etc.

Of course, any sane person would ask, "Haven't we made things more complicated?". The answer to this is (and this is where the genius of this method comes in): $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$ is easier to do than $n - a_3^2$ because a_3^2 requires us to square some big ugly number (e.g. 2680), whereas we have already calculated both $(n - a_2^2)$ (which is 443856 in the example below) and $\{b_2(2a_2 + b_2)\}$ (which is 422400 in the example below).

Subtracting 443856 – 422400, is easier than squaring 2680!!!!!

Example: $\sqrt{7203856}$ (once again!):

	n	7203856	our first estimate (a_1) is 2000.
	a_1^2	– 4000000	
<u>step #1</u>	**$n-a_1^2$**	3203856	$= b_1(4000 + b_1) \rightarrow$ <u>$b_1 = 600$</u>
	$b_1(2a_1 + b_1)$	– 2760000	←
<u>step #2</u>	**$n-a_2^2$**	443856	$= b_2(5200 + b_2) \rightarrow$ <u>$b_2 = 80$</u>
	$b_2(2a_2 + b_2)$	– 422400	←
<u>step #3</u>	**$n-a_3^2$**	21456	$= b_3(5360 + b_3) \rightarrow$ <u>$b_3 = 4$</u>
	$b_3(2a_3 + b_3)$	– 21456	←
		0	So our answer is *exactly* **<u>2684</u>**

2) Calculate each square root using the *Short Algebraic Method.* It is important that you do the problem and organize your work exactly like the example given above. Notice that the first three problems are the same ones given in the previous exercise. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$ d) $\sqrt{393129}$ e) $\sqrt{145924}$

$\sqrt{\ }$ Algorithm – Sheet #7

The Square Root Algorithm (with zeroes)

- This method is basically identical to the *Short Algebraic Method*, but it cuts out all the unnecessary writing, and there is an added shortcut that aids us in determining the values for $2a_1$, $2a_2$, $2a_3$, etc. This new shortcut is as follows: With our example of $\sqrt{7203856}$, the values for $2a_1$, $2a_2$, $2a_3$ are 4000, 5200, 5360. The first of these values is found simply by doubling a_1, which is 2000·2 = 4000. *The rest of these values are found by taking the previous value and adding the new b value to it two times*. So from 4000, we add b_1, which is 600, giving us 4600, and then add 600 again, giving us our next value, 5200. From 5200, we add b_2, which is 80, giving us 5280, and then adding 80 again, gives us our next value, 5360.

Example: $\sqrt{7203856}$ (once again!)

Step #1: We know that $a_1 = 2000$, so we write down 2000 twice. Multiplying the two 2000's gives us the 4000000 that is written under 7203856, and subtracting, we get 3203856. Then we add 2000 plus 2000 to get 4000, but we put a box in place of the zeroes, and another box underneath the first box. So at this point, everything looks like this:

2000	7203856
2000	– 4000000
4□	3203856
□	

It is important to understand that the boxes represent b_1. So at this point, with both the short and long algebraic method, we had had this equation: $3203856 = b_1(4000 + b_1)$, and we asked ourselves, "what must b_1 be so that $b_1(4000 + b_1)$ is less than 3203856?" Here, with this new method, we are asking essentially the same thing. We need to fill in the two boxes with the same value (i.e. the value for b_1). And this value must be a certain number of hundreds – resulting in a product of 4100·100, or 4200·200, or 4300·300, etc. Since 4700·700 is bigger than 3203856, we put 600 in the two boxes, and write the product of 4600·600, which is 2760000, under 3203856.

Step #2: We now add the left column (4600 + 600), which gives us 5200, and subtract the right column (3203856–2760000), which is 443856. Once again, we write a box in place of the zeroes of 5200, and another box under that one. Everything now looks like this:

2000	7203856
2000	– 4000000
4600	3203856
600	– 2760000
52□	443856
□	

Similarly to step #1, we need to put the same number (which is the tens' place of our final answer) into both boxes so that the resulting product is less than 443856. The possibilities are 5210·10, or 5220·20, or 5230·30, etc. Since 5290·90 is a bit too big, we put 80 into both boxes, and write the product of 5280·80, which is 422400, under 443856.

Step #3: We add the left column and subtract the right column, resulting in 5360 and 21456, respectively. We put a box in place of the zero in 5360, and a box below it (which is *not* shown below). 4 can be put into both boxes, resulting in 5364·4, which is *exactly* 21456. The end result, is that all of our work looks like this (quite short, actually!):

2000	7203856
2000	– 4000000
4600	3203856
600	– 2760000
5280	443856
80	– 422400
5364	21456
4	– 21456
	0

- The answer, 2684, comes from the underlined digits.
- A remainder of zero tells us that our answer is exact.
- This method of the square root algorithm is slightly different from what is done in 8th grade.

(Continued on next page →)

1) Do each problem three times. First using the *long algebraic method*; secondly, using the *short algebraic method*; and lastly, using the *square root algorithm* as described in the example above.

a) $\sqrt{725904}$ b) $\sqrt{72361}$ c) $\sqrt{28761769}$

2) Do each problem using the *square root algorithm* only.

a) $\sqrt{665856}$ b) $\sqrt{6041764}$

$\sqrt{\ }$ Algorithm – Sheet #8

Calculate each square root using only the *square root algorithm*, as shown on the previous sheet.

1) $\sqrt{2116}$

3) $\sqrt{413449}$

5) $\sqrt{18395521}$

2) $\sqrt{327184}$

4) $\sqrt{683929}$

6) $\sqrt{86620249}$

Algebra – Sheet #1

Formulas

1) At Bob Rent-a-Car, the rates are \$28/day and 8¢/mile.

a) What is the formula that is used to calculate the cost (C)? (Hint: D for the number of days and M for the number of miles.)

Use the above formula to calculate the cost for renting a car at Bob Rent-a-Car for…

b) 10 days and 900 miles.

c) 7 days and 345 miles.

2) *Galileo's Law of Falling Bodies* is given by the formula: $\mathbf{D = 16T^2}$, where D is the number of feet an object falls (neglecting air resistance) after being dropped for T seconds. Find the distance that an object falls after being dropped for…

a) 3 seconds.

b) 5½ seconds

Signed Numbers

Simplify by combining the signed numbers. If you get stuck, think of a checking account.

3) $6 - 2 - 17$

4) $-5 + 12$

5) $12 - 5$

6) $4 - 6$

7) $-6 + 4$

8) $-10 - 3 - 6$

9) $10 + 3 + 6$

10) $-10 - 3 - 6 + 10 + 3 + 6$

11) $-3 + 8 - 12 - 5 + 4$

12) $8 + 4 - 3 - 12 - 5$

13) $9 - 2 - 6 + 15 - 11$

14) $-2 - 6 - 11 + 9 + 15$

Expressions

Simplify by combining like terms.

15) $3X + 7X$

16) $4X + 9X$

17) $20X - 5X$

18) $3X + 6X + 4X$

19) $2X + 8X + 6X + 13X$

20) $2X + 8Y + 6X + 13Y$

Equations

Solve each equation, by finding the value for X that makes the equation balance.

21) $X = 21 + 8$

22) $X + 7 = 10$

23) $X = 13 - 8$

24) $X - 4 = 20$

25) $X = 14 \cdot 3$

26) $5 \cdot X = 35$

27) $5X = 35$

28) $X = 42 \div 7$

29) $X \div 3 = 12$

Algebra – Sheet #2

Formulas

1) At Bob Rent-a-Car (see previous sheet), what is the cost of renting a car for 20 days and 700 miles?

2) Find the distance that an object falls after being dropped for 4 seconds.

3) *Gauss's Formula* for summing together a sequence of numbers is: $\mathbf{S = \frac{N}{2} \cdot (F+L)}$, where F is the first number, L is the last number, and N is the number of numbers.

Find the sum of each sequence of numbers.

a) 40+41+…+50

b) 300+301+…+700

Signed Numbers

Simplify by combining the signed numbers. If you get stuck, think of a checking account.

4) $5 - 8$

5) $-8 + 5$

6) $13 - 9$

7) $-9 + 13$

8) $-3 + 8 - 2 - 7 + 9$

9) $7 - 18 + 10 - 13 + 4$

10) $-7 - 5 - 9$

11) $-9 + 100$

12) $46 - 70$

13) $-46 - 70$

14) $\frac{3}{8} - \frac{7}{8}$

15) $-\frac{3}{8} + \frac{4}{5}$

16) $-\frac{5}{9} - \frac{4}{7}$

Expressions

Simplify by combining like terms.

17) $5X + 9X$

18) $9X + 5X$

19) $6X - 4X$

20) $-4X + 6X$

21) $3X + 5X + 6X$

22) $6X + X$

23) $3X - 5X$

24) $8X + 4Y + 6Y - 3X$

25) $8X + 4 + 6 - 3X$

Equations

Solve each equation, by finding the value for X that makes the equation balance.

26) $X = 5 \cdot 3$

27) $6X = 24$

28) $X + 12 = 15$

29) $X + 12 = 7$

30) $X - 10 = 4$

31) $X - 10 = -4$

32) $X - 10 = -14$

Algebra – Sheet #3

Formulas

1) Find the sum of $213+214+\ldots+262$.

2) The *temperature conversion formulas* are:

$$C = \frac{5}{9}\cdot(F - 32)$$

$$F = \frac{9}{5}\cdot C + 32$$

Use these formulas to…

a) Convert 25°C to Fahrenheit.

b) Convert 30°C to Fahrenheit.

c) Convert 86°F to Celsius.

d) Convert 50°F to Celsius.

Signed Numbers

Simplify.

3) $-7 + 9$

4) $11 - 19$

5) $-19 + 11$

6) $9 - 20 - 4 + 33 - 7$

7) $-9 - 15 - 2$

8) $\frac{2}{5} - \frac{2}{3}$

9) $-\frac{3}{13} - \frac{5}{13}$

10) $(5)(9)$

11) $(-5)(-9)$

12) $(-5)(9)$

13) $(5)(-9)$

14) $(-2)(15)$

15) $(-6)(-7)$

16) $(3)(-10)$

17) $(-120)(-110)$

Expressions

Simplify by combining like terms.

18) $6X + 21X$

19) $2X - 7X$

20) $-7X + 2X$

21) $-4X - 6X$

22) $3X + 5Y + 6X$

23) $X + X$

24) $4Y + 8X - Y - 13X$

25) $-9X - 4 - 12 - 3X$

Equations

Solve each equation. Always check that your answer is correct.

26) $X + 1 = 7$

27) $X - 1 = 7$

28) $5X = 45$

29) $X \div 4 = 20$

30) $X - 10 = -6$

31) $X + 5 = -3$

32) $X \div 3 = 12$

33) $2X = 11$

34) Use Guess and Check!
$2X + 5 = 19$

Algebra – Sheet #4

Formulas

1) Convert 104°F to Celsius.

2) Convert 5°C to Fahrenheit.

3) Convert 5°F to Celsius.

4) Convert 12°C to Fahrenheit.

5) Convert –13°F to Celsius.

6) Convert –20°C to Fahrenheit.

7) Convert –40°F to Celsius.

Signed Numbers

Simplify.

8) $-9+13$

9) $-9-13$

10) $23-32$

11) $-32+23$

12) $(3)(6)$

13) $(3)(-6)$

14) $3-6$

15) $(-3)(+6)$

16) $-3+6$

17) $(-3)(-6)$

18) $-3-6$

19) $(-15)\div(-3)$

20) $(15)\div(-3)$

21) $\frac{15}{-3}$

22) $7--4$

23) $5-+9$

24) $-4--6$

25) $-7-(-3-5)$

Expressions

Simplify by combining like terms.

26) $5X+7X$

27) $3A+6B-8A$

28) $9+5X-4$

29) $Y+4+X-12-5X+Y$

30) $3X-73+10X$

31) $5X+13-5X-2$

32) $5X-8Y-X+6$

33) $-7+X-8-4X$

One-Step Equations

Solve each equation by getting X alone. Show what is done to each side. Check your answers with the previous sheet.

Example:
$$\begin{array}{r} X+7=10 \\ \underline{-7\quad -7} \\ \boxed{X\ =\ 3} \end{array}$$

34) $X+1=7$

35) $X-1=7$

36) $5X=45$

37) $X\div4=20$

38) $X-10=-6$

39) $X+5=-3$

40) $X\div3=12$

41) $2X=11$

Algebra – Sheet #5

Formulas (See previous sheets for formulas.)

1) At Bob Rent-a-Car, what is the cost of renting a car for 15 days and 1000 miles?

2) Find the distance that an object falls after being dropped for 1½ seconds.

3) Convert 20°C to Fahrenheit.

4) Convert 20°F to Celsius.

5) Find the sum of 8+9+10+…+67.

Signed Numbers
Simplify.

6) $-2+9$
7) $-5+1$
8) $-5-1$
9) $19-33$
10) $(5)(-7)$
11) $5-7$
12) $(-3)(8)$
13) $-3+8$
14) $(-4)(-3)$
15) $-4-3$
16) $(14)\div(-2)$
17) $(-14)\div(2)$
18) $\frac{-14}{2}$
19) $(-14)\div(-2)$
20) $\frac{-14}{-2}$
21) $10--8$
22) $9-+2$
23) $9+-2$
24) $-12--5$
25) $-7--3+-5-+4$

Expressions
Simplify by combining like terms.

26) $X+X+X+A+A$
27) $7A-5Y+8-A+12Y$
28) $4+X-5B-5X+7+X$
29) $X-6Y+5-X-5+8Y$
30) $8X-7-13X-8$
31) $-3X-2-X+9$
32) $-X+5+6X-8$

One-Step Equations
Solve each equation by getting X alone. Show what is done to each side. Check that your answers are correct.

33) $X+5=8$
34) $X+12=8$
35) $X-7=10$
36) $X-8=-2$
37) $3X=21$
38) $X\div 10=7$
39) $X+9=-4$
40) $X\div 5=9$
41) $2X=8$
42) $\frac{1}{2}X=8$

Algebra – Sheet #6

Formulas

Euclid's Formula for Perfect Numbers is given by the formula:

$P = (2^{(N-1)})\cdot(2^N - 1)$,

where P is a perfect number only if $(2^N - 1)$ is a prime number.

1) Calculate the first four perfect numbers by using the above formula, and putting in N=2, and then N=3, and then N=4, etc. Don't forget to check that $(2^N - 1)$ is prime. (Show your work on a separate sheet.)

Signed Numbers

Simplify.

2) $-5 - 10$

3) $-4 + 12$

4) $-9 + 4$

5) $6 - 14$

6) $(-5)(-8)$

7) $(-5)(3)$

8) $(-18)\div(-6)$

9) $\frac{-18}{-6}$

10) $(18)\div(-6)$

11) $\frac{35}{-5}$

12) $-4 - -10$

13) $-5 - +8$

14) $-12 + 25 - 6 - 3$

15) $5 - -6 + -8 - +7$

Expressions

Simplify by combining like terms.

16) $7X - 8 + 2X$

17) $7X - 8 + 2$

18) $7 - 3X + 5 - 6X$

19) $X - 5 + 5X + 2$

20) $7 - X - 3 + 6X - 1$

21) $-7X + 9 + 5X - 2X$

22) $Y - 7 + 8 - 4A - 3Y + X$

Solving Equations

Solve each equation by getting X alone. Show what is done to each side. Check that your answers are correct.

23) $5X = 40$

24) $5X = -40$

25) $-5X = -40$

26) $-5 + X = 40$

27) $5X + 1 = 3X + 9$

28) $7X + 5 = 4X + 26$

29) $5X - 7 = X + 3$

30) $8X + 19 = 3$

31) $-7X + 4 = -31$

32) $X-7+6X = 8-X+9$

Algebra – Sheet #7

Signed Numbers

Simplify.

1) $-8 + 13$

2) $20 - 50$

3) $(5)(-9)$

4) $(-4)(-6)$

5) $(-28)\div(-4)$

6) $(16)\div(-2)$

7) $\frac{16}{-2}$

8) $-5 \cdot \frac{-3}{5}$

9) $\frac{3}{4} \cdot \frac{-16}{27}$

10) $5 - -9$

11) $-8 - 2 + 6 - 7 + 4$

12) $-5 + -9 - +7 - -2$

Expressions

Simplify by combining like terms.

13) $7X + 6 - 12X$

14) $-6 - 7X - 8$

15) $-4X - 5 - X + 10$

16) $8B + 4A - B - 4$

Solving Equations

Solve each equation by getting X alone. Show what is done to each side. Check that your answers are correct.

17) $4X = 28$

18) $3X = -21$

19) $X + 6 = 2$

20) $X - 4 = 11$

21) $-4X = 12$

22) $X \div 3 = 15$

23) $\frac{X}{3} = 15$

24) $-8X = -4$

25) $5X - 4 = 2X + 23$

26) $2X + 11 = 9X - 3$

27) $6X - 5 + 2X = 17 + 15X - 77$

28) $4X - 5 - X - 3 = -2X + 4 + 9X$

29) *Challenge!* $-18X - 5 + X + 4 + 38X - X - 5 = 3X - 18 - 5X + 30 + 9X - 5$

Algebra – Sheet #8

Signed Numbers

Simplify.

1) $-8-3$

2) $34-42$

3) $(4)(-7)$

4) $(-8)(-3)$

5) $(40)\div(-4)$

6) $(-20)\div(-5)$

7) $\frac{-20}{-5}$

8) $6\cdot\frac{7}{-15}$

9) $\left(-\frac{4}{5}\right)\cdot\left(-\frac{5}{6}\right)$

10) $-7--10$

11) $-6+9+4-7$

12) $-2--7+-8$

Solving Equations

Solve each equation by getting X alone. Show what is done to each side. Check that your answers are correct.

13) $-5X=-40$

14) $X+7=-2$

15) $6X=-42$

16) $X\div4=8$

17) $\frac{X}{4}=8$

18) $7X-21=3X-9$

19) $8X+3-5X=7-4X-32$

20) $X-8-6X=-7+X-3$

21) $6X-7=2X-10$

22) *Challenge!*
$\frac{1}{6}X+\frac{2}{3}-\frac{3}{4}X=-\frac{7}{10}+\frac{2}{3}X-\frac{2}{5}$

23) *Challenge!*
$7X+4-X-8-11X-14=-12+49X+23-11-52X$

24) *Challenge!*
$-X-2\frac{2}{3}-12X+13+5X-5\frac{1}{2}=13\frac{2}{3}X+5-\frac{3}{4}X-21\frac{1}{6}-17\frac{5}{12}X$